# On a mean reverting dividend strategy with Brownian motion

Benjamin Avanzi[a], Bernard Wong[a]

[a]*Actuarial Studies, Australian School of Business*
*UNSW Sydney NSW 2052, Australia*

**Abstract**

In actuarial risk theory, the introduction of dividend pay-outs in surplus models goes back to Bruno de Finetti (1957). Dividend strategies that can be found in the literature often yield pay-out patterns that are inconsistent with actual practice. One issue is the high variability of the dividend payment rates over time. We aim at addressing that problem by specifying a dividend strategy that yields stable dividend pay-outs over time.

We model the surplus of a company at time $t$,

$$U(t) = x + \mu t + \sigma W(t), \quad t \geq 0, \tag{1}$$

with a standard Brownian motion $\{W(t)\}$. In this framework, Gerber (1972) and Gerber and Shiu (2004) considered a barrier strategy, Gerber et al. (1981) a linear barrier strategy (with increasing slope) and Asmussen and Taksar (1997) a threshold strategy. In model (1) with a classical definition of ruin (absorbed at a level 0), the dividend strategy that maximises the present value of dividends until ruin is a barrier strategy; see Gerber (1972) and Gerber and Shiu (2004).

In this paper, dividends are paid continuously at a rate $g > 0$ of the current (modified) surplus. Hence, the surplus of the company $X(t)$ after distribution of dividends has dynamics

$$dX(t) = [\mu - gX(t)]\, dt + \sigma dW(t), \quad t \geq 0, \tag{2}$$

with $X(0) = x$. This is an Ornstein-Uhlenbeck process (with additional drift $\mu$). The drift of (2) at time $t$,

$$\mu - gX(t), \tag{3}$$

is positive whenever the surplus is below a certain level $l = \mu/g$, and negative when the surplus exceeds that level. The surplus is thus reverting around $l$, and the dividend pay-out rate is itself reverting around an average (or mean) rate $g \cdot l = \mu$; hence justifying its name of *mean reverting* dividend strategy. This 'average' pay-out rate is equal to the original drift of (1). Here $X(t)$ operates as a buffer reservoir to yield a smoother dividend flow with target annual rate $\mu$, irrespective of the value for $g$. Alternatively, $g$ can be interpreted as a target *return on investment* (ROI), achieved by an adaptation of the surplus level to $l$ (function of $\mu$).

We determine the distribution of the present value of dividends when the surplus process is never absorbed. After introducing an absorbing barrier $a$ (inferior to the initial surplus) and stating the Laplace transform of the time of absorption, we derive the expected present value of dividends until absorption. The latter is then also determined if dividends are not paid whenever the surplus is too close to the absorbing barrier. The calculation of the optimal value of the parameter $l$ (and equivalently $g$) is discussed. We conclude by comparing both barrier and mean reverting dividend strategies.

The paper can be downloaded from SSRN: `http://ssrn.com/abstract=1504401`.

*Key words:* Dividends, Brownian motion, Ornstein-Uhlenbeck process, Mean reverting

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*Email address:* `b.avanzi@unsw.edu.au` (Benjamin Avanzi)